COT 3100 In-class Exercise 5

Name: USF ID:

Problem 1 Prove the following statements by direct proof.

1. For all integers and, if and then.

Proof:

Suppose that and are any particular but arbitrarily chosen integers such that and. [We must show that].

By definition of divisibility, and for some integersand. Then,

by substitution

by algebra

Let. Then is an integer since products and difference of integers are integers and 2, 3, and are integers. Therefore,

where is an integer.

Thus, divides by definition of divisibility. [This is what was to be shown.]

1. A necessary condition for an integer to be divisible by 6 is that it be divisible by 2.

Rewrite the given statement formally as:

Proof:

Suppose that is any particular but arbitrarily chosen integers such that is divisible by 6. [We must show that is divisible by 2].

By definition of divisibility, for some integers. Then,

by algebra

Let. Then is an integer since products of integers are integers and 3 and are integers. Therefore,

where is an integer.

Thus, is divisible by 2 by definition of divisibility. [This is what was to be shown.]

1. For all integers, if then.

Proof:

Suppose that is any particular but arbitrarily chosen integers such that. [We must show that].

By the quotient-remainder theorem, can be written in the form for some integer. Then,

by substitution

by algebra

Let. Then is an integer since products and sums of integers are integers and 1, 5, 6 and are integers. Therefore,

where is an integer.

Therefore, by the quotient-remainder theorem. [This is what was to be shown.]

1. For all integers and, if and, then.

Proof:

Suppose that are any particular but arbitrarily chosen integers such that. [We must show that].

By the quotient-remainder theorem, can be written in the form and for some integer. Then,

by substitution

by algebra

Let. Then is an integer since products and sums of integers are integers and 1, 2, 5, and are integers. Therefore,

where is an integer.

Therefore, by the quotient-remainder theorem. [This is what was to be shown.]

1. For any integer, is not divisible by 4.

Proof:

Suppose that is any particular but arbitrarily chosen integer. By the quotient-remainder theorem, can be written in one of the forms or for some integer.

Case 1 ( for some integer):

by substitution

by algebra

Let. Then is an integer since products of integers are integers and 1 and are integers. Therefore, is not divisible by 4 by the quotient-remainder theorem.

Case 2 ( for some integer):

by substitution

by algebra

Let. Then is an integer since products of integers are integers and 1 and are integers. Therefore, is not divisible by 4 by the quotient-remainder theorem.

Hence, in both cases, is not divisible by 4 [as was to be shown].

1. For any odd integer,

Proof:

Suppose that is any particular but arbitrarily chosen odd integer. By definition of odd, for some integer. Then,

by substitution

by algebra

Let is an integer because products and sums of integers are integers and 1 and are integers. And , So, by definition of floor, the left-hand side of the equation to be shown is .

The right-hand side of the equation to be shown is by substitution and algebra

Since both the left-hand and right-hand sides equal , they are equal to each other. That is,. [This is what was to be shown.]